

Solving Complex Fuzzy Linear System of Equations by using QR-Decomposition Method

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Abstract— In this paper, QR-decomposition method for solving the complex fuzzy linear equation $Cz = w$ in which C is a crisp complex matrix and w is an arbitrary complex fuzzy vector is considered. Some examples are given to illustrate the proposed method.

Keywords— Complex fuzzy numbers, Fuzzy linear systems, Fuzzy approximate solutions, QR-decomposition.

I. INTRODUCTION

Many real-world engineering systems are too complex to be defined in precise terms, imprecision is often involved in any engineering design process. Fuzzy systems have an essential role in this fuzzy modeling, which can formulate uncertainty in actual environment. In many linear systems, some of the system parameters are vague or imprecise, and fuzzy mathematics is a better tool than crisp mathematics for modeling these problems, and hence solving a fuzzy linear system [11] or a fuzzy differential equation is becoming more important. The concept of fuzzy numbers and arithmetic operations with these numbers were first introduced and investigated by Zadeh [22]. A different approach to fuzzy numbers and the structure of fuzzy number spaces was given by Puri and Ralescu [16], Goetschell and Voxman [12] and Wu and Ma Ming [20].

Since Friedman et al. proposed a general model for solving an $n \times n$ fuzzy linear systems whose coefficients matrix is crisp and the right-hand side is an arbitrary fuzzy numbers vector by an embedding approach, many works have been done about how to deal with some advanced fuzzy linear systems such as dual fuzzy linear systems (DFLS), general fuzzy linear systems (GFLS), full fuzzy linear systems (FFLS), dual full fuzzy linear systems (DFFLS) and general dual fuzzy linear systems (GDFLS). These works were performed mainly by Allahviranloo et al. [2,3,4,5], Abbasbandy et al. [1,7], Zheng et al. [1,9] and Dehgham et al. [10] and so on. In traditional fuzzy linear systems, the uncertain elements were usually denoted by the parametric form of fuzzy numbers. Based on arithmetic operations of the fuzzy number, the fuzzy linear systems could be extended into crisp function linear systems. Therefore the solutions of the fuzzy linear systems can be obtained by solving the model by means of ordinary analytical and numerical methods.

However, very few researchers have developed methods to solve fuzzy complex system of linear equations. The fuzzy complex numbers was introduced by J.J.Buckley in 1989 [9]. The $n \times n$ fuzzy complex linear systems have been studied by M.A. Jahanigh [15]. Solution of fuzzy complex linear system of linear equations was described by Rahgooy et al. [18] and applied to circuit analysis problem. In 2014, Behera and Chakraverty [8] discussed the fuzzy complex system of linear equations by adding and subtracting the left and right bounds of the fuzzy complex unknowns and the right-hand side is fuzzy complex vector.

In this paper complex fuzzy linear system is investigated. A numerical procedure for calculating the fuzzy solution is designed. Finally, some examples are given to illustrate our method. The structure of this paper is organized as follows:

In Section 2, we recall the triangle fuzzy number and present the concept of the complex fuzzy linear systems. The computing model to the complex fuzzy linear systems is proposed in detail and the fuzzy approximate solution of the complex fuzzy linear systems is obtained by solving the crisp systems of linear equations using QR- decomposition method. Some examples are given to illustrate our method in Section 4 and the conclusion is drawn in Section 5.

II. PRELIMINARIES

There are several definitions for the concept of fuzzy numbers.

2.1 The fuzzy number

Definition 2.1. A fuzzy number is a fuzzy set like $u: R \rightarrow I = [0,1]$ which satisfies:

- (1) u is upper semi-continuous,
- (2) u is fuzzy convex, i.e., $u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$ for all $x, y \in R, \lambda \in [0,1]$,
- (3) u is normal, i.e., there exists $x_0 \in R$ such that $u(x_0) = 1$,
- (4) $\text{supp } u = \{x \in R \mid u(x) > 0\}$ is the support of the u , and its closure $\text{cl}(\text{supp } u)$ is compact.

Let E^1 be the set of all fuzzy numbers on R .

Definition 2.2. A fuzzy number u in parametric form is a pair $(\underline{u}, \bar{u})$ of functions $\underline{u}(r), \bar{u}(r), 0 \leq r \leq 1$, which satisfies the requirements:

- (1) $\underline{u}(r)$ is a bounded monotonic increasing left continuous function,
- (2) $\bar{u}(r)$ is a bounded monotonic decreasing left continuous function,
- (3) $\underline{u}(r) \leq \bar{u}(r), 0 \leq r \leq 1$.

A crisp number x is simply represented by $(\underline{u}(r), \bar{u}(r)) = (x, x), 0 \leq r \leq 1$. By appropriate definitions the fuzzy number space $\{\underline{u}(r), \bar{u}(r)\}$ becomes a convex cone E^1 which could be embedded isomorphically and isometrically into a Banach space.

Definition 2.3 Let $x = (\underline{x}(r), \bar{x}(r)), y = (\underline{y}(r), \bar{y}(r)), 0 \leq r \leq 1$, and $k \in R$. Then

- (1) $x = y$ iff $\underline{x}(r) = \underline{y}(r), \bar{x}(r) = \bar{y}(r)$,
- (2) $x + y = (\underline{x}(r) + \underline{y}(r), \bar{x}(r) + \bar{y}(r))$,
- (3) $x - y = (\underline{x}(r) - \underline{y}(r), \bar{x}(r) - \bar{y}(r))$,
- (4) $kx = \begin{cases} (k\underline{x}, k\bar{x}), k \geq 0 \\ (k\bar{x}, k\underline{x}), k < 0 \end{cases}$,

Definition 2.4 An arbitrary fuzzy complex number may be represented as $X = p + iq$, where $p = (\underline{p}(r), \bar{p}(r))$ and $q = (\underline{q}(r), \bar{q}(r))$ for all $0 \leq r \leq 1$ are two real fuzzy number. Hence one may have $X = (\underline{p}(r), \bar{p}(r)) + i(\underline{q}(r), \bar{q}(r))$.

Definition 2.5 For any two arbitrary complex fuzzy numbers $x = p + iq, y = u + iv$ where p, q, u, v are fuzzy numbers, their arithmetic is as follows:

- (1) $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$;
- (2) $kx = kp + ikq, k \in R$;

$$(3) \quad z_1 \times z_2 = \{(x_1 \times x_2) - (y_1 \times y_2)\} + i\{(x_1 \times y_2) + (y_1 \times x_2)\}.$$

2.2 Complex fuzzy linear systems

Definition 2.6. The linear system equation

$$\begin{cases} c_{11}z_1 + c_{12}z_2 + \dots + c_{1n}z_n = w_1 \\ c_{21}z_1 + c_{22}z_2 + \dots + c_{2n}z_n = w_2 \\ \dots \\ c_{n1}z_1 + c_{n2}z_2 + \dots + c_{nn}z_n = w_n \end{cases} \quad (2.1)$$

where $c_{ij}, 1 \leq i, j \leq n$ are complex numbers and $w_i, 1 \leq i, j \leq n$ are complex fuzzy numbers, is called a complex fuzzy linear system (CFLS).

Using matrix notation, we have

$$Cz = w \quad (2.2)$$

A complex fuzzy numbers vector

$$z = (z_1, z_2, \dots, z_n)^T$$

is called a fuzzy solution of the complex fuzzy linear system (2.1) if z satisfies (2.2).

III. SOLVING COMPLEX FUZZY LINEAR SYSTEM OF EQUATION

3.1 Equivalent fuzzy linear system

Theorem 3.1. The $n \times n$ complex fuzzy linear system (2.1) is equivalent to a $2n \times 2n$ order fuzzy linear system

$$Gx = b \quad (3.1)$$

where

$$G = \begin{pmatrix} A & -B \\ B & A \end{pmatrix}, x = \begin{pmatrix} p \\ q \end{pmatrix}, b = \begin{pmatrix} u \\ v \end{pmatrix} \quad (3.2)$$

Proof. We denote $C = A + iB, A, B \in R^{n \times n}$ and $w = u + iv$ where u, v are fuzzy number vectors. We also suppose the unknown vector $z = p + iq$ where p, q are two unknown fuzzy number vectors.

Since $Cz = w$, we have

$$(A + iB)(p + iq) = u + iv$$

i.e.

$$(Ap - Bq) + i(Aq + Bp) = u + iv.$$

Comparing with the coefficient of i , we have

$$\begin{cases} Ap - Bq = u \\ Aq + Bp = v \end{cases}$$

i.e.

$$\begin{pmatrix} A & -B \\ B & A \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} u \\ v \end{pmatrix},$$

It admits a $2n$ order fuzzy linear system.

We express it in matrix form as follow:

$$Gx = b$$

3.2 The model

When z, w of fuzzy linear equation (2.3) are denoted by the parametric form i.e.

$$w = [w_1, w_2, \dots, w_n]^T, w_j = u_j + iv_j = [\underline{u}_j(r), \bar{u}_j(r)] + i[\underline{v}_j(r), \bar{v}_j(r)],$$

$$z = [z_1, z_2, \dots, z_n]^T, z_j = p_j + iq_j = [\underline{p}_j(r), \bar{p}_j(r)] + i[\underline{q}_j(r), \bar{q}_j(r)], j = 1, 2, \dots, n, 0 \leq r \leq 1.$$

Some results for solving fuzzy linear equation (3.2) are obtained by the following analysis.

Theorem 3.2. The fuzzy linear equation (3.2) can be extended to a crisp function linear system as follow

$$SX(r) = Y(r) \quad (3.3)$$

Where

$$S = \begin{pmatrix} G^+ & -G^- \\ -G^- & G^+ \end{pmatrix}, X(r) = \begin{pmatrix} \underline{x}(r) \\ -\bar{x}(r) \end{pmatrix}, Y(r) = \begin{pmatrix} \underline{b}(r) \\ -\bar{b}(r) \end{pmatrix} \quad (3.4)$$

in which the elements g_{ij}^+ of matrix G^+ and g_{ij}^- of matrix G^- are determined by the following way: if

$$g_{ij} \geq 0, g_{ij}^+ = g_{ij} \text{ else } g_{ij}^+ = 0, 1 \leq i, j \leq n; \text{ if } g_{ij} < 0, g_{ij}^- = g_{ij} \text{ else } g_{ij}^- = 0, 1 \leq i, j \leq n.$$

Proof. Let $b = [\underline{b}(r), \bar{b}(r)], 0 \leq r \leq 1$ and $x = [\underline{x}(r), \bar{x}(r)]$. Suppose $G = G^+ + G^-$ in which the elements g_{ij}^+ of matrix G^+ and g_{ij}^- of matrix G^- are determined by the following way: if $g_{ij} \geq 0, g_{ij}^+ = g_{ij} \text{ else } g_{ij}^+ = 0, 1 \leq i, j \leq n$; if $g_{ij} < 0, g_{ij}^- = g_{ij} \text{ else } g_{ij}^- = 0, 1 \leq i, j \leq n$. The fuzzy linear equation $Gx = b$ can be expressed as

$$(G^+ + G^-)[\underline{x}(r), \bar{x}(r)] = [\underline{b}(r), \bar{b}(r)].$$

Since

$$kx_j = \begin{cases} (k\underline{x}_j(r), k\bar{x}_j(r)), k \geq 0 \\ (k\bar{x}_j(r), k\underline{x}_j(r)), k < 0 \end{cases}$$

We have

$$Gx = \begin{cases} (G\underline{x}(r), G\bar{x}(r)), G \geq 0 \\ (G\bar{x}(r), G\underline{x}(r)), G < 0. \end{cases}$$

So the equation (3.4) be rewritten as

$$G^+[\underline{x}(r), \bar{x}(r)] + G^-[\underline{x}(r), \bar{x}(r)] = [G^+\underline{x}(r), G^+\bar{x}(r)] + [G^-\bar{x}(r), G^-\underline{x}(r)],$$

$$[G^+\underline{x}(r) + G^-\bar{x}(r), G^+\bar{x}(r) + G^-\underline{x}(r)] = [\underline{b}(r), \bar{b}(r)].$$

Thus we have

$$\begin{cases} G^+ \underline{x}(r) + G^- \bar{x}(r) = \underline{b}(r) \\ G^+ \bar{x}(r) + G^- \underline{x}(r) = \bar{b}(r) \end{cases}$$

or

$$\begin{cases} G^+ \underline{x}(r) - G^- (-\bar{x}(r)) = \underline{b}(r) \\ G^+ (-\bar{x}(r)) - G^- \underline{x}(r) = -\bar{b}(r) \end{cases}$$

Expressing in matrix form , we have

$$\begin{pmatrix} G^+ & -G^- \\ -G^- & G^+ \end{pmatrix} \begin{pmatrix} \underline{x}(r) \\ -\bar{x}(r) \end{pmatrix} = \begin{pmatrix} \underline{b}(r) \\ -\bar{b}(r) \end{pmatrix}. \quad \square$$

Remark 3.1 If $CZ = w$, then $\underline{z} + \bar{z}$ and $\underline{z} - \bar{z}$ are the solutions of complex fuzzy linear system, i.e.,

$$C(\underline{z} + \bar{z}) = (\underline{w} + \bar{w}) \text{ and } C(\underline{z} - \bar{z}) = (\underline{w} - \bar{w}).$$

Remark 3.2 If $CZ = w$, then $\underline{z} - \bar{z}$ is the solution of complex fuzzy linear system, i.e., $C(\underline{z} - \bar{z}) = (\underline{w} - \bar{w})$ and

$$|C|(\underline{z} - \bar{z}) = (\underline{w} - \bar{w}).$$

In order to solve the fuzzy system of linear equation (3.2), we need to consider the systems of linear equations (3.4). It seems that we have obtained the minimal solution of the fuzzy linear system (3.2) as

$$X(r) = S^\dagger Y(r) \quad (3.5)$$

i.e.,

$$\begin{pmatrix} \underline{x}(r) \\ \bar{x}(r) \end{pmatrix} = \begin{pmatrix} G^+ & -G^- \\ -G^- & G^+ \end{pmatrix}^\dagger \begin{pmatrix} \underline{b}(r) \\ -\bar{b}(r) \end{pmatrix}. \quad (3.6)$$

where $S^\dagger$ is the Moore-Penrose generalized inverse of matrix S .

Definition 3.1. Let $X(r) = (\underline{x}_j(r), -\bar{x}_j(r)), 1 \leq j \leq 2n$ denotes the minimal solution of (2.5). The fuzzy number vector $Z = [\underline{p}_j(r), \bar{p}_j(r)] + i[\underline{q}_j(r), \bar{q}_j(r)], 1 \leq j \leq n$ defined by

$$\begin{aligned} \underline{p}_j(r) &= \min\{\underline{x}_j(r), \bar{x}_j(r), \underline{x}_j(1), \bar{x}_j(1)\}, \\ \bar{p}_j(r) &= \max\{\underline{x}_j(r), \bar{x}_j(r), \underline{x}_j(1), \bar{x}_j(1)\}, \\ j &= 1, 2, \dots, n, 0 \leq r \leq 1, \end{aligned} \quad (3.7)$$

$$\begin{aligned} \underline{q}_j(r) &= \min\{\underline{x}_j(r), \bar{x}_j(r), \underline{x}_j(1), \bar{x}_j(1)\}, \\ \bar{q}_j(r) &= \max\{\underline{x}_j(r), \bar{x}_j(r), \underline{x}_j(1), \bar{x}_j(1)\}, \\ j &= n+1, n+2, \dots, 2n, 0 \leq r \leq 1. \end{aligned} \quad (3.8)$$

is called the fuzzy minimal solution of the fuzzy linear systems (2.3). If $\underline{p}_j(r), \bar{q}_j(r), 1 \leq j \leq 2n$ are all fuzzy numbers

then $z = \{[\underline{p}_j(r), \bar{p}_j(r)] + i[\underline{q}_j(r), \bar{q}_j(r)], 1 \leq j \leq n\}$ is called a strong complex fuzzy minimal solution of the complex fuzzy linear systems (2.1). Otherwise, z is called weak complex fuzzy minimal solution.

3.3 QR-decomposition method

Theorem 3.3 If eq.(3.4) $S \in m \times n$, then there exist an orthogonal $Q \in m \times n$ and an upper triangular $R \in m \times n$ so that $S = QR$.

Proof. Suppose $n = 1$ and that Q is Householder matrix so that if $R = Q^T S$ then $R(2:m) = 0$. It follows that $S = QR$ is a QR factorization of S . For general n we partition S ,

$$S = [S_1 | v]$$

Where $v = S(:, n)$.

By induction, there exists an orthogonal $Q_1 \in R^{m \times m}$ so that $R_1 = Q_1^T S_1$ is upper triangular. Set $w = Q_1^T v$ and let $w(n:m) = Q_2 R_2$ be the QR factorization of $w(n:m)$. If

$$Q = Q_1 \begin{bmatrix} I_{n-1} & 0 \\ 0 & Q_2 \end{bmatrix}$$

Then

$$A = Q \left[R_1 \begin{array}{c} w(1:n-1) \\ R_2 \end{array} \right]$$

is a QR factorization of S .

Lemma 3.1 If A is an $m \times n$ matrix with full column rank, then A can be factored as $A = QR$, where Q is an $m \times n$ matrix whose column vectors form an orthonormal basis for the column space of A and R is a $n \times n$ invertible upper triangular matrix.

Theorem 3.4. If Eq.(2.5) S is a matrix with full column rank, and if $S = QR$ is a QR -decomposition of A , then the system for $SX = Y$ can be expressed as

$$RX = Q^T Y \quad (3.11)$$

and the solution can be expressed as

$$X = R^{-1} Q^T Y \quad (3.12)$$

Proof. According to Lemma 3.1., we have

$$S = QR$$

Where Q is $4m \times 4n$ orthogonal matrix, R is a $4n \times 4n$ invertible upper triangular matrix. The fact that Q has orthonormal column implies that $Q^T Q = I$, so multiplying both side of $S = QR$ by on the left side

$$R = Q^T S \quad (3.13)$$

Therefore, the system for $SX = Y$ can be expressed as

$$RX = Q^T Y.$$

In order to obtain the solution, due to R is invertible upper triangular matrix, so

$$X = R^{-1} Q^T Y.$$

3.4 A sufficient condition for strong fuzzy solution

The key point to make the solution matrix being a strong fuzzy solution is that $S^\dagger Y(r)$ is fuzzy matrix, i.e., each element in which is a triangular fuzzy number. By the analysis, it is equivalent to the condition $S^\dagger \geq 0$.

Theorem 3.5. If

$$((G^+ - G^-)^\dagger + (G^+ + G^-)^\dagger) \geq 0, ((G^+ - G^-)^\dagger - (G^+ + G^-)^\dagger) \geq 0,$$

the complex fuzzy linear equation(2.1) has a strong complex fuzzy minimal solution as follows:

$$z = [\underline{p}_j(r), \bar{p}_j(r)] + i[\underline{q}_j(r), \bar{q}_j(r)], 0 \leq r \leq 1 \quad (3.10)$$

Where

$$\begin{cases} \underline{x}(r) = \begin{pmatrix} \underline{p}(r) \\ \underline{q}(r) \end{pmatrix} = E \underline{b}(r) - F \bar{b}(r) \\ \bar{x}(r) = \begin{pmatrix} \bar{p}(r) \\ \bar{q}(r) \end{pmatrix} = -F \underline{b}(r) + E \bar{b}(r) \\ E = \frac{1}{2}((G^+ - G^-)^\dagger + (G^+ + G^-)^\dagger) \\ F = \frac{1}{2}((G^+ - G^-)^\dagger - (G^+ + G^-)^\dagger) \end{cases}$$

Proof. The proof is straight forward

IV. NUMERICAL EXAMPLE

In this section, we will demonstrate the efficiency and superiority of the proposed method using numerical examples.

Example 4.1 Consider the following CFSLE:

$$\begin{cases} \tilde{z}_1 - \tilde{z}_2 = (r, 2-r) + i(1+r, 3-r) \\ \tilde{z}_1 + 3\tilde{z}_2 = (4+r, 7-2r) + i(r-4, -1-2r) \end{cases} \quad (4.1)$$

Let $\tilde{z}_1 = \underline{p}_1 + i \underline{q}_1 = (p_1, \bar{q}_1) + i(q_1, \bar{q}_1)$, $\tilde{z}_2 = \underline{p}_2 + i \underline{q}_2 = (p_2, \bar{q}_2) + i(q_2, \bar{q}_2)$. The extend maxtrix is

$$S = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 \end{pmatrix}$$

And Rank(S)=8, so we obtain

$$Q = \begin{pmatrix} -0.7071 & 0.6396 & -0.2023 & -0.2236 & 0 & 0 & 0 & 0 \\ -0.7071 & -0.6396 & 0.2023 & 0.2236 & 0 & 0 & 0 & 0 \\ 0 & -0.4264 & -0.6068 & -0.6708 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.7416 & 0.6708 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.7071 & 0.6396 & -0.2023 & -0.2236 \\ 0 & 0 & 0 & 0 & -0.7071 & -0.6396 & 0.2023 & 0.2236 \\ 0 & 0 & 0 & 0 & 0 & -0.4264 & -0.6068 & -0.6708 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.7416 & 0.6708 \end{pmatrix}$$

$$R = \begin{pmatrix} -1.4142 & -2.1213 & 0 & -0.7071 & 0 & 0 & 0 & 0 \\ 0 & -2.3452 & -0.4264 & 0.6396 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.3484 & -2.4271 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.7889 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1.4142 & -2.1213 & 0 & -0.7071 \\ 0 & 0 & 0 & 0 & 0 & -2.3452 & -0.4264 & 0.6396 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.3484 & -2.4271 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.7889 \end{pmatrix}$$

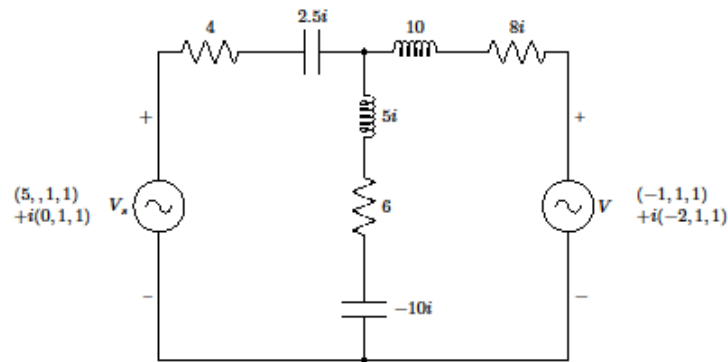
Now by using (3.2), we have

$$\begin{pmatrix} \underline{p_1} \\ \underline{p_2} \\ \underline{q_1} \\ \underline{q_2} \\ -\underline{p_1} \\ -\underline{p_2} \\ -\underline{q_1} \\ -\underline{q_2} \end{pmatrix} = R^{-1}Q^T \begin{pmatrix} r \\ 4+r \\ 1+r \\ r-4 \\ r-2 \\ 2r-7 \\ r-3 \\ 2r+1 \end{pmatrix} = \begin{pmatrix} 1.375+0.625r \\ 0.875+0.125r \\ 0.125+0.625r \\ -1.375+0.125r \\ -0.2875+0.875r \\ -1.375+0.375r \\ -1.625+0.875r \\ 0.875+0.375r \end{pmatrix}.$$

The solution may be written as

$$\begin{pmatrix} \tilde{z_1} \\ \tilde{z_2} \end{pmatrix} = \begin{pmatrix} (1.375+0.125r, 2.875-0.875r) + i(0.125+0.625r, 1.625-0.875r) \\ (0.875+0.125r, 1.375-0.375r) + i(-1.375+0.125r, -0.875-0.375r) \end{pmatrix}$$

Example 4.2 Consider a simple RLC circuit with fuzzy current and fuzzy source in Figure 1. The CFSLE for the circuit is as follow :



$$\begin{cases} (10 - 7.5i) \tilde{z}_1 - (6 - 5i) \tilde{z}_2 = (4 + r, 6 - r) + i(-1 + r, 1 - r) \\ -(6 - 5i) \tilde{z}_1 + (16 + 3i) \tilde{z}_2 = (-2 + r, -r) + i(-3 + r, -1 - r) \end{cases} \quad (4.2)$$

Let $\tilde{z}_1 = \tilde{p}_1 + i\tilde{q}_1 = (\underline{p}_1, \overline{p}_1) + i(\underline{q}_1, \overline{q}_1)$, $\tilde{z}_2 = \tilde{p}_2 + i\tilde{q}_2 = (\underline{p}_2, \overline{p}_2) + i(\underline{q}_2, \overline{q}_2)$. The extend matrix is

$$S = \begin{pmatrix} 10 & 0 & 7.5 & 0 & 0 & 6 & 0 & 5 \\ 0 & 16 & 0 & 0 & 6 & 0 & 5 & 3 \\ 0 & 5 & 10 & 0 & 7.5 & 0 & 0 & 6 \\ 5 & 3 & 0 & 16 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 5 & 10 & 0 & 7.5 & 0 \\ 6 & 0 & 5 & 3 & 0 & 16 & 0 & 0 \\ 7.5 & 0 & 0 & 6 & 0 & 5 & 10 & 0 \\ 0 & 0 & 6 & 0 & 5 & 3 & 0 & 16 \end{pmatrix}.$$

And we obtain

$$Q = \begin{pmatrix} -0.6785 & 0.0383 & -0.2174 & 0.3857 & -0.0239 & 0.3954 & 0.4249 & 0.0629 \\ 0 & -0.8876 & 0.1659 & 0.3010 & 0.2183 & -0.0736 & -0.0590 & 0.1938 \\ 0 & -0.2774 & -0.7365 & -0.1883 & 0.0879 & 0.2058 & -0.2931 & -0.4573 \\ -0.3392 & -0.1473 & 0.2181 & -0.7970 & 0.3458 & 0.1555 & 0.1656 & 0.0966 \\ 0 & -0.3382 & 0.0622 & -0.2472 & -0.8283 & -0.0528 & 0.3188 & -0.1839 \\ -0.4071 & 0.0230 & -0.1699 & 0.0013 & 0.1107 & -0.8671 & 0.1014 & -0.1747 \\ -0.5088 & 0.0287 & 0.2804 & 0.0160 & -0.2873 & 0.0628 & -0.7580 & -0.0169 \\ 0 & 0 & -4730 & -0.1694 & -0.2090 & -0.1146 & -0.1272 & 0.8213 \end{pmatrix}$$

$$R = \begin{pmatrix} -14.7394 & -1.0177 & -7.1238 & -9.7019 & 0 & -13.1281 & 7.1238 & -3.3923 \\ 0 & -18.0268 & -2.3715 & -3.7792 & -10.7340 & 0.7411 & -7.5305 & -4.1354 \\ 0 & 0 & -12.0838 & 4.9730 & -6.2713 & -4.0395 & 5.4090 & 12.5773 \\ 0 & 0 & 0 & -13.8874 & -2.9250 & 1.9066 & -4.9705 & -1.0086 \\ 0 & 0 & 0 & 0 & -7.3583 & -0.4347 & -5.9185 & -2.2809 \\ 0 & 0 & 0 & 0 & 0 & -11.5309 & 0.7967 & 1.1572 \\ 0 & 0 & 0 & 0 & 0 & 0 & -4.4908 & -1.8456 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 11.3242 \end{pmatrix}$$

Now by using (3.2), we have

$$\begin{pmatrix} \underline{p_1} \\ \underline{p_2} \\ \underline{q_1} \\ \underline{q_2} \\ -\underline{p_1} \\ -\underline{p_2} \\ -\underline{q_1} \\ -\underline{q_2} \end{pmatrix} = R^{-1}Q^T \begin{pmatrix} 4+r \\ -2+r \\ -1+r \\ -3+r \\ r-6 \\ r \\ r-1 \\ r+1 \end{pmatrix} = \begin{pmatrix} 0.3164+0.0378r \\ 0.0347+0.0307r \\ 0.0708+0.0378r \\ -0.2380+0.0307r \\ -0.3920+0.0378r \\ -0.0961+0.0307r \\ -0.1464+0.0378r \\ 0.1765+0.0307r \end{pmatrix}.$$

The solution may be written as

$$\begin{pmatrix} \tilde{z_1} \\ \tilde{z_2} \end{pmatrix} = \begin{pmatrix} (0.3164+0.0378r, 0.3920-0.0378r) + i(0.0708+0.0378r, 0.1464-0.0378r) \\ (0.0347+0.0307r, 0.0961-0.0307r) + i(-0.2380+0.0307r, -0.1765-0.0307r) \end{pmatrix}$$

V. CONCLUSION

In this work we presented a model for solving complex fuzzy linear equation $Cz = w$ where C is a crisp complex matrix and w is an arbitrary complex fuzzy vector, respectively. The complex fuzzy linear system is converted to a high order linear system $SX(r) = Y(r)$. We use the QR-decomposition of the coefficient matrix S to obtain fuzzy solution of complex fuzzy linear systems. In addition, numerical examples showed that our method is feasible to solve this type of complex fuzzy linear systems.

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