

Optimal design of steel and composite vessels with tube branch joint

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Abstract - This paper presents a unified methodology of combining heuristic fuzzy design and FEM verification to design optimally vessels with a branch joint. This method is applicable to steels and composites. Background for this study is the recognized need for constructing vessels with branch tubes needed for processing liquids and gases with minimal ecological and maintenance problems. The methodology of fuzzy optimum design is used. The goals and constraints are expressed in a consistent formulation. First, design variables like wall thickness are defined discretely within ranges. Then decision variables are formulated like cost and safety factors. The total goal is maximization of the end user satisfaction on the design. It is product of satisfaction functions of decision variables. The stresses are calculated by reasonable free body models and notch factors. Two steels are studied, a basic low strength steel (LS) and a high strength steel (HS). At low pressure $p=0.1$ MPa the LS vessel is 4.2 times more satisfactory than the HS vessel. At higher pressure $p=1$ the LS vessel is 0.4 times less satisfactory than HS vessel. An analytical stress result agrees reasonably with FEM results at a test pressure. The optimal choice depends not only on economics and technology but also on the societal and environmental changes and megatrends. This methodology can be used to explore novel concepts.

Keywords: pressure vessel, fuzzy optimum design, steels, fatigue.

I. INTRODUCTION

This study is motivated by the **recognized global need** for constructing vessels with branch joints needed for processing liquids and gases with minimal ecological problems. Cylindrical vessels with elliptical ends are reasonable choices.

Customers require a reliable lifetime of tens of years with minimal initial cost, maintenance and still stand costs. These goals can be formulated and solved by applying heuristic fuzzy optimum design with FEM verification.

There are many choices for vessel materials. All materials are composite from macro to microstructural level. In certain corrosive loadings fiber reinforced composites (FRP) are a competitive choice to highly alloyed steel and in others not. In this task a reasonable selection of steel and composite materials are options.

Composites for structural engineering are commonly made by combining materials from five commonest material groups, metals, ceramics glasses, plastics and polymers. Even only two components one can obtain many useful combinations of the properties. The one with largest volume fraction is called matrix. Others can be in form of fibres, platelets and globular forms.

The fibre reinforced plastics are designed to combine the strength of strong fibres in desired direction, chemical, static and fatigue strength, impact, environmental and thermal endurance. Composite material design is discussed by [1] (Agarwal, Broutman, 1990), by [2] (Barbero, 1990) and [3] (Swanson, 1997). A novel generalized failure criterion is proposed by [4] (Knops, 2008). Now a modified criterion is developed.

Basic vessel theory and designs of steel materials are discussed by [5] (Harvey, 1991). The main method of vessel design is the basic shell theory. Mandatory design rule codes are based on theory, tests. FEM is used to verify the analytical results.

The vessel design goal can be expressed easily as fuzzy satisfaction of the end user. This method is based on results of [6] (Diaz, 1988). [7] (Martikka, Pöllänen, 2009) have applied multi-objective optimisation by technical laws and heuristics to vessel design. This same methodology is applicable to a wide range of optimization tasks. Design of metal and non-metal materials for pressure vessels is discussed by [8] (Martikka, Taitokari, 2011).

In these tasks the reliability based design can be applied as discussed by [9] (Leitch R.D, 1988), and [10] (Dhillon, and Singh, 1981). Optimum design of FRP vessels is considered by [11] (Martikka, Taitokari 2006) under variable pressure loads. Design of FRP vessels with internal dynamical load due to mixing rotor is discussed by [12] (Taitokari, Martikka, 2009). External dynamic load to vessels are often seismic and wind loadings. Optimal design of seismically loaded vessels is discussed by [13] (Martikka, Pöllänen, 2011). Endurance dimensioning of steels are discussed by [14] (Gurney, 1978) and [15] (Meyer, 1985).

Design of welded tube to vessel joint for liquid gas is discussed by [16] (Martikka, Taitokari, 2012). Welded beam joint design is discussed by [17] (Martikka, Pöllänen, 2008). Optimal dynamical and statics design of steel bridges is discussed by [18] (Martikka, Taitokari, 2013).

The main purpose of this study is to develop methodology by which branched steel vessels can be optimised using fuzzy optimum design with goal of maximization of user satisfaction.

II. DESIGN APPROACH

The objective in this section is to present an overview of the present approach for obtaining a satisfactory design. Composites offer more design variables than steels allowing a trade-off use of many decision variables, like low cost, low weight, high safety at extreme loadings.

The goalsof designing of thin shelled structures for process industry

Successful producers are able to recognize the needs of customer. The top level goal is psychological, to obtain maximal satisfaction of the customer on the total costs and reliable time of utilization of a product. The task of the designer is to devise a method and apply it to reach the goal.

A. Satisfaction on technology of the design concept

This is mainly decided by the design engineers. Technical decision variables are factors of safety.

B. Satisfaction on cost is mainly defined by the customer.

It includes the total cost, maintenance and recycling and ecology costs. The total goal is maximal satisfaction on technology and total life cycle costs.

III. THE OBJECT OF DESIGN

The objective in this section is to present an overview of the object of the present case study.

3.1 A typical vessel structure

In Figure 1 typical structures and loadings are shown.

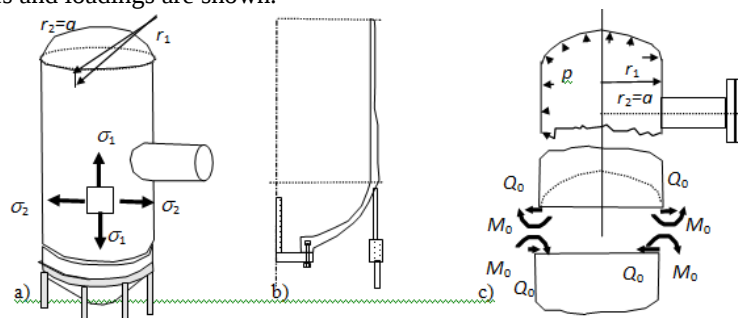
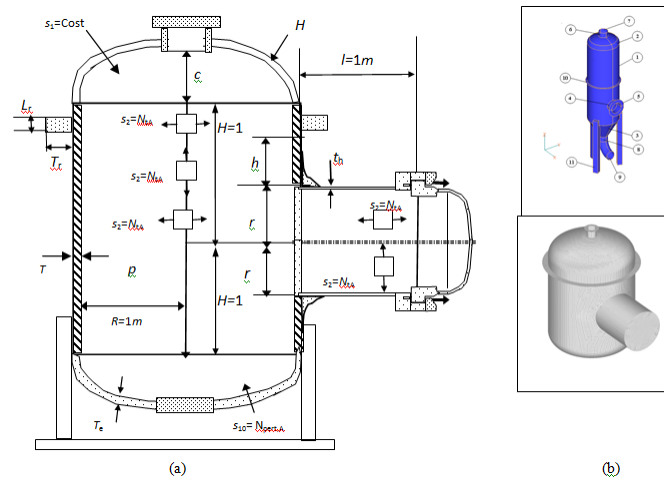


FIGURE 1: AN OVERVIEW SKETCH OF VESSEL; A) DOMINANT STRESSES AND MAIN RADII; B) BOTTOM DESIGNS; C) SURROGATE MODELING OF INTERNAL STRESSES

The basic geometry of the structure is shown in Figure 2a. The typical features are support, bottom opening, and top opening and side branch opening. The main load is now internal pressure p inside. Branch tube may be closed by two options: a conventional model and a quick closure model. The critical locations are shown.

Reasonable dimensions are: Geometry of the main vessel A : $R = 1$ m, $H = 1$ m, tube branch $r = .5$ m, $l = 1$ m. Reasonable choices for the $p = 1$ MPa case are $T = 0.013$ m, $t_h = 0.005$ m, $T_e = 0.013$ m $T/T_r = 1.2, T/L_r = 1$ whence $T_r = T/1.2 = 0.013/1.2$, $L_r = T = 0.013$ m.

A typical industrial vessel is shown in Figure 2b.



**FIGURE 2: A) THE VESSEL IN THIS CASE STUDY.
B) A TYPICAL INDUSTRIAL VESSEL AND FEM MODEL IN THIS STUDY.**

3.2 Joint stresses

The complex bending stresses at the joint can be estimated using a simple surrogate bent beam model shown in Figure 3 schematically. The corresponding FEM model is shown in Figure 4.

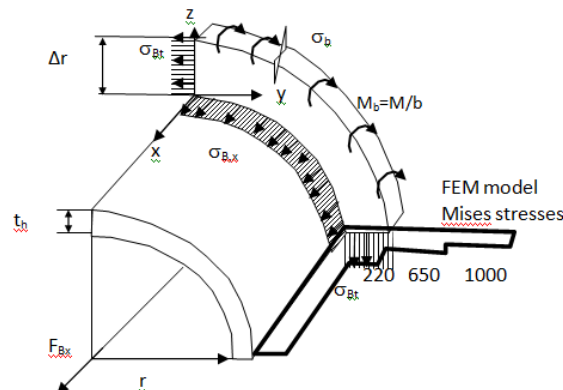


FIGURE 3: A SIMPLE BENT BEAM MODEL FOR ESTIMATING THE JOINT STRESSES .

The bending force on the model is F_{Bx} . It is one quarter of the resultant pressure force acting on the branch head plate. It causes a bending moment. The lever is some radial flange distance Δr . The width of the beam is peripheral length around the tube B of thickness t_h .

$$\sigma_b = \frac{M}{W} = \frac{F_{Bx} \Delta r}{\frac{1}{6} b t_h^2}, b = 2\pi \frac{1}{4}, F_{Bx} = F_8 = p\pi^2 \frac{1}{4} \quad (1)$$

Substitution gives the nominal bending stress of B at joint

$$\sigma_b = \frac{M}{W} = \frac{p\pi^2 \frac{1}{4} \cdot \Delta r}{\frac{1}{6} 2\pi \frac{1}{4} \cdot t_h^2} = \frac{pr}{2t_h} \cdot 6 \frac{\Delta r}{t_h} = \pm \sigma_{Bx} 6 \frac{\Delta r}{t_h} = S_{x,b}, \quad \tilde{x} = \frac{\Delta r}{t_h} \quad (2)$$

Here $\tilde{x}$ = extent of stiffening influence by this fictive model.

The axial nominal stress at the shell B is caused by the pressure

$$S_{x,n} = \sigma_{B,x} = \frac{pr}{2t_h} = \frac{1MPa \cdot 0.5m}{2 \cdot 0.010m} = 25MPa \quad (3)$$

The total summed nominal axial stress in tube B is sum

$$S_x = S_{x,n} + S_{x,b} = \sigma_{Bx} \left(1 + 6 \frac{\Delta r}{t_h} \right) = \sigma_{Bx} (1 + 6\tilde{x}) \quad (4)$$

The nominal stress in the y direction of the branch B is

$$S_y = \sigma_{Bt} = 2\sigma_{Bx} \quad (5)$$

The equivalent Mises stress at B is

$$\sigma_{VAB} = \sqrt{S_x^2 + S_y^2 - S_x S_y} = \sigma_{Bx} \sqrt{(1 + 6\tilde{x})^2 + 4 - 2 \cdot (1 + 6\tilde{x})} = \sigma_{Bx} G(\tilde{x}) \quad (6)$$

The peak stress is obtained by multiplying it by a stress concentration factor K_t

$$\sigma_{VABmax} = K_t \sigma_{VAB} \quad (7)$$

This is a heuristic tentative model. The following rough estimates are feasible

$$\sigma_{VAB}(x) = \sigma_{xB} \sqrt{3 + (6\tilde{x})^2} = G \sigma_{xB} = G \frac{pr}{2t_h} \quad (8)$$

The G is a rough model only

$$\tilde{x} = 0.2 \Rightarrow G \approx 2, \quad \tilde{x} \approx 0.4 \Rightarrow G \approx 3, \quad \tilde{x} \approx 0.6 \Rightarrow G \approx 4$$

The stress concentration is estimated in the program using choices $K_t = 3$ and $G = 3$

$$\sigma_{VABmax} = K_t \sigma_{VAB} = K_t \cdot G \cdot \sigma_{xB} = K_{total} \cdot \sigma_{xB} = 3 \cdot 3 \cdot 25MPa = 225MPa \quad (9)$$

The stress distribution with FEM is shown in Figure 9b. The notch is sharp giving a high stress concentration of about 10 since this is a linear elastic FEM element model.

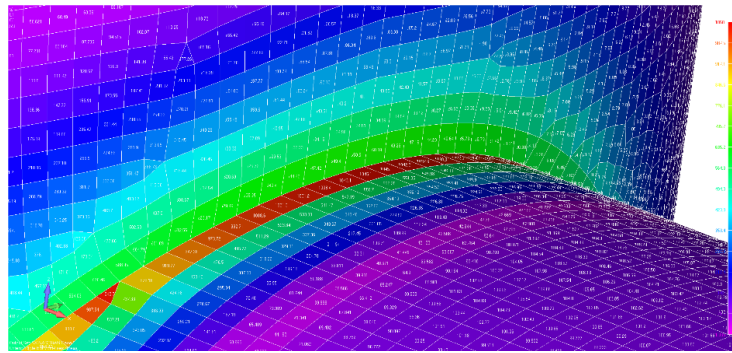


FIGURE 4: THE MISES STRESS BY FEM [20] AT THE JOINT.

FEM gives maximum value of Mises stress as 1045 MPa.

The stress concentration by FEM is $K_{FEM} = 10$ due to a very small notch radius

$$\sigma_{VABmax}(FEM, sharp) = K_{FEM} \cdot \sigma_{xB} \approx 10 \cdot 100MPa = 1000MPa \quad (10)$$

If the FEM model notch geometry is changed to have a moderate notch radius then the stress concentration will be acceptably small

$$\sigma_{VABmax}(FEM, smooth) = \tilde{K}_{FEM} \cdot \sigma_{xB} \approx 3 \cdot 100MPa = 300MPa \quad (11)$$

In the analytical model the total stress concentration is a heuristic estimate,

$$\sigma_{VAB_{max}} = K_t \cdot G \cdot \sigma_{xB} = 3 \cdot 3 \cdot 25MPa = 225MPa \dots 4 \cdot 4 \cdot 25MPa = 400MPa \quad (12)$$

The FEM and analytical stress peaks are fairly close.

The mean Mises stress and amplitude Mises stresses are

$$\sigma_{Vm} = \sigma_{VAB,max} \quad , \quad \sigma_{Va} = 0.2\sigma_{Vm}, \quad \Delta\sigma = 2\sigma_{Va} \quad (13)$$

Here the assumption is made that the amplitude stress is 0.2 times the mean stresses. These are used in fatigue life estimation

3.3 Simplified analysis of stresses in a tube branch connection

The objective in this section is to present an overview of the present approach to get the tube branch stresses using a simplified analysis based on FBD balance shown in Figure 5.

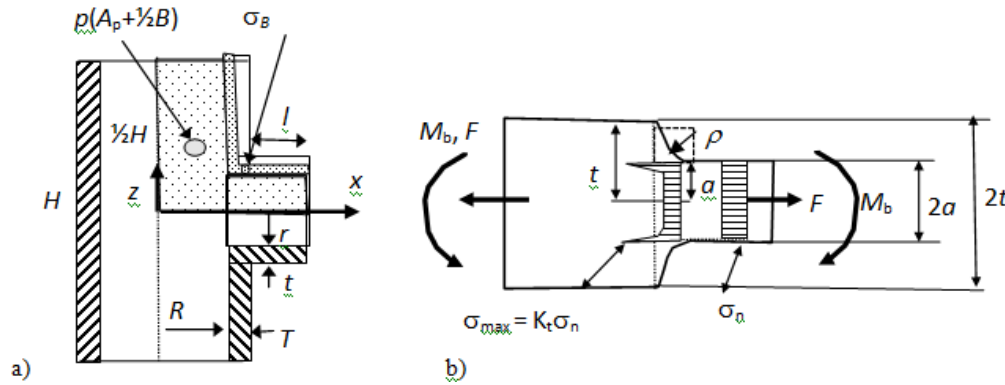


FIGURE 5: SIMPLIFIED ANALYSIS OF STRESSES IN A TUBE BRANCH CONNECTION;
A) SKETCH; B) NOTCH FACTOR MODEL

The pressure force is balanced by the resultant force due to shell tangential stress.

The nominal tangential stress acting on the cross sectional area of the nozzle tube is

$$\sigma_B = p \left[\frac{A_p}{B} + \frac{1}{2} \right], A_p = \frac{1}{2} HR + lr, B = lt + \left(\frac{1}{2} H - r \right) T, \sigma_{B,max} = K_t \sigma_B \quad (14)$$

Here the stress areas are A_p and B , $\sigma_{B,max}$ is the maximal stress and K_t is the notch factor.

Here the notch factor is estimated with an approximation of the model in [19] (Dubbel, 1994).

Flat plate surrogate model is used to estimate the stress concentration factor.

Two options are given for model parameter q , $q=0.5$ for bending $q=1$ for tension.

$$K_{t,stepped,plate} = 1 + \frac{1}{G} = 1 + q \sqrt{\frac{a}{\rho}} \quad (15)$$

The tube wall may be about $t_h=5..8$ mm. Then $a=0.5 t_h$. Radius may be $\rho = 1$ mm. The estimated range is

$$K_{t,stepped,plate} = 1 + q \sqrt{\frac{4mm}{1mm}} = 1 + 2q = 2(q=0.5) \dots 3(q=1) \quad (16)$$

IV. FUZZY GOAL FORMULATION USING DECISION VARIABLES

The objective in this section is to present of an overview of the present approach to get a satisfactory design. The design variable vector $\mathbf{x}$ = (load functions, geometry, materials) is the tool.

Using them the decision variable event vector $\mathbf{s}$ = (cost, factors of safety...) = $\mathbf{s}(\mathbf{x})$ is formulated. They are defined fuzzily (Diaz, 1988). The total event is decision variable s and it is intersection of the chosen decision variables s_k

$$s = s_1 \cap s_2 \cap s_3 \cap s_4 \cap s_5 \cap s_6 \quad (17)$$

The design goal is maximization of the total satisfaction of the customer on the product

$$P(s) \Rightarrow P(s) = P(s_1) \cdot P(s_2) \cdot \dots \cdot P(s_n), Q = \max P \quad (18)$$

Now all goals and constraints are formulated consistently by one flexible fuzzy function. This is illustrated in Figure 11

V. DESIGN VARIABLES

The aim in this section is to present the main steel options with appropriate property data and estimations. Composite materials can also be activated with their appropriate criteria.

5.1 Material design variables

Steel option design variables are shown in Table 1. The basic steel S52 is commonly used in steel vessel constructions. The OX steel is used when high yield strength is needed. The stainless steels 904L is used when corrosion resistance is needed. An alternative to it is FRP. The E25Cr is used in corrosion resistant castings.

TABLE 1.

STEEL MATERIAL VARIABLES. THE PARIS LAW C AND M PARAMETERS ARE CALCULATED WITH GURNEY'S MODEL[14] (GURNEY, 1978). STRESS RATIO $R_s = \sigma_{MIN} / \sigma_{MIN} = 0$, $K_{TH} (N \cdot MM^{3/2})$,

	OXsteel im=6	St52 steel im=7	Stainless 904L im=3	E25Cr steel im=4
Yield strength (MPa)	Re(6) = 1000	Re(7) = 335	Re(3) = 220	Re(4) = 500
Tensile strength (MPa)	Rm(6) = 1100	Rm(7) = 500	Rm(3) = 490	Rm(4) = 700
unit cost (eur/kg)	Cm(6) = 20	Cm(7) = 5	Cm(3) = 5	Cm(4) = 5
density (kg/m ³)	rho(6) = 8000	rho(7) = 8000	rho(3) = 8000	rho(4) = 8000
ecological value	eco(6) = .5	eco(7) = .5	eco(3) = .7	eco(4) = .1
corrosion resistance	corres(6) = .5	corres(7) = .5	corres(3) = .8	corres(4) = .9
Elastic modules (MPa)	E(6) = 205000	E(7) = 205000	E(3) = 205000	E(4) = 205000
Threshold intensity(Nmm ^{-3/2})	Kth(6) = 275	Kth(7) = 190-144 · Rs	Kth(3) = 3MPam ^{1/2}	Kth(4) = 3MPam ^{1/2}
Initial crack size (mm)	a0(6) = 1	a0(7) = 1	a0(3) = 1	a0(4) = 1
Paris C parameter	C(6) = 4.64E-12	C(7) = 1.67E-14	C(3) = 1.67E-14	C(4) = 1.67E-14
Paris m exponent	m(6) = 2.52	m(7) = 3.36	m(3) = 3.36	m(4) = 3.36

5.2 Functional design variables and parameters

The pressure p was now one input parameter. It can be changed to a variable if optimal loading is sought.

5.3 Geometrical design variables

Here the design variables related to the vessel and branches are

T = wall thickness with discrete values in the range 0.005...0.013m, wall thickness h of the branch tube with range 0.005...0.013 and relative height c/R with range 0.45...0.65. The end wall thickness ratio T_e/T was set to 1.

$$T = T(iT), iT = 1 \dots 8 \quad h = th(ith), ith = 1 \dots 8, \quad xcR(ixcR) = \frac{c}{R}, ixcR = 1 \dots 8 \quad (19)$$

The design variables related to the stiffener rings are. $E_r = E_z$ for FRP and $E_r = E$ for steels

The ranges are for $x_1 = 0.5 \dots 1.4$ and for $x_2 = 1 \dots 2$ and for $x_3 = 1 \dots 2.5$

$$x_1 = \frac{E}{E_r} = xEr(ixEr), \quad x_2 = \frac{T^2}{A_r} = \frac{T}{L_r} \frac{T}{T_r} = xAr(ixAr), \quad x_3 = \frac{T(iT)}{R} \quad (20)$$

VI. DECISION VARIABLES FOR DEFINING GOALS

The objective in this section is to present of overview of the decision variables used.

Decision variables $s(k)$ depend on the design variables $x(i)$, $s = s(x)$.

The results for the optimal low alloy vessel at load $p = 1$ MPa are the following: The optimal geometry is $T = 0.013$ m, $t_h = 0.01$, $T_e = 0.013$, $x_{CR} = c/R = 0.45$, $x_{eT} = 1$, $x_{Er} = 1$, $x_{Ar} = 2$. Satisfaction $P(s_k)$ and s_k are obtained for the optimal low alloy construction

6.1 Goal of obtaining low cost K or decision variable s_1

Cost is now the material cost of the shells and plates.

6.1.1 The ellipsoidal end of the vessel

The bottom radius is the same as the radius of the main tube A. Now R is given and height c of the ellipsoid is a design variable. The radii of curvature of the shell are

$$r_2 = a = b = R, \quad r_1 = \frac{c^2}{R}, \quad c = x_{CR} \cdot R \quad (21)$$

Radius of curvature at the calotte at $x = 0, z = c$

$$r_2(x, z) = \left[\left(\frac{R}{c} \right)^4 z^2 + x^2 \right]^{\frac{1}{2}} \Rightarrow r_{calot} = r_2(0, c) = \frac{R^2}{c} \quad (22)$$

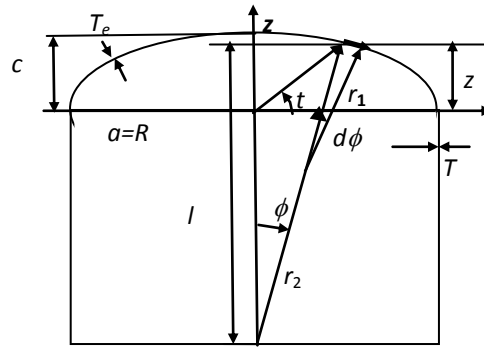


FIGURE 6: GEOMETRY OF THE ELLIPSOIDAL CALOTTE END OF THE VESSEL

The total cost the first decision variable. Same unit cost C is assumed for all components

$$s_1 = K = C(m_A + 2m_e + m_B + m_{Be}) \quad (23)$$

6.2 Goal of obtaining satisfactory safety factor for the main shell in tangential direction

Tangential direction is chosen as the strong direction L. The branch opening raises the stress

$$s_2 = N_{tA} = \frac{R_{mt}}{\sigma_{tA}}, \quad \sigma_{tA} \Rightarrow \frac{pRH}{(H-r)T} \rightarrow \frac{pR}{T} \cdot \frac{1}{1-\frac{r}{H}} = \sigma_L \quad (24)$$

6.3 Goal of obtaining satisfactory safety factor for the main shell A in axial direction

The decision variable is now the safety factor against axial stress σ_{xA}

$$s_3 = N_{xA} = \frac{R_{mx}}{\sigma_{xA}}, \quad \sigma_{xA} = \frac{p \frac{1}{4} \pi R^2}{TR(\frac{1}{2} \pi - \phi)} = \frac{3}{4} \frac{pR}{T} = \sigma_T, \quad \phi = \frac{\pi}{6} \quad (25)$$

6.4 Goal of obtaining satisfactory safety factor for the tube shell B in tangential direction

Tangential direction is now chosen as the strong direction L and the load stress σ_{tB} are

$$s_5 = N_{tB} = \frac{R_{mt}}{\sigma_{tB}}, \sigma_{tB} = \sigma_{t5} = \frac{pr}{t_h} \quad (26)$$

6.6 Goal of obtaining satisfactory safety factor for the calotte of the main shell

Safety factor of the calotte of the calotte of the main shell A

$$s_7 = N_{calot} = \frac{R_{mx}}{\sigma_{calot}}, r_{calot} = \frac{R^2}{c}, T_{calot} = T_e, \sigma_{calot} = \frac{pr_{calot}}{T_{calot}} \quad (27)$$

Here R_{mx} is UTS of FRP in x direction, in steel it is $R_m(im)$

6.6 Goal of obtaining satisfactory safety factor for joint of the shells based on Mises equivalent stress

The decision variable is the safety factor

$$s_8 = N_{AB} = \frac{R_m(im)}{\sigma_{VAB,max}} \quad (28)$$

6.7 Goal of obtaining satisfactory safety factor for joint of the shells using the simple method

The decision variable is

$$s_9 = N_{xB} = \frac{R_m}{\sigma_{xB,max}} \quad (29)$$

The maximal stress is

$$\sigma_{xB,max} = K_t \sigma_B, K_t \approx 2 \quad (30)$$

6.8 Goal of obtaining satisfactory safety factor for the main shell maximal stress location

The decision variable is factor of safety

$$s_{10} = N_{pert,A} = \frac{R_m}{\sigma_{V,pertA}} \quad (31a)$$

The nominal stress S and the perturbation stress acting on vessel A are

$$S = \frac{pR}{2T} = \frac{1 \cdot 1}{2 \cdot 0.013} = 38MPa, \quad r_1 = 0.3R, \quad e = 0.146, \quad \sigma_{V,pertA} = 1.1S \quad (31b)$$

6.9 Goal of obtaining satisfactory safety factor against buckling of the main shell

The pressure loading at low pressure causes a buckling risk.

To avoid buckling risk the tangential stress should be tensile and the factor of safety within a range

$$s_{11} = N_{buckl,A} = \frac{\sigma_{cr}}{\sigma_2}, \quad \sigma_{cr} = 0.605E \frac{T}{R} \quad (32)$$

Here σ_{cr} is The buckling strength for a compressed ideal cylinder with wall T and radius R

The principal stresses are

$$\sigma_1 = \sigma_\phi = \frac{pR}{2T_e}, \quad \sigma_\theta = \sigma_2 = \frac{pR}{T_e} \left(1 - \frac{R^2}{2c^2} \right) = \frac{pR}{T_e} \left(1 - \frac{R}{2r_1} \right) \quad (33)$$

6.10 Goal of obtaining long enough crack initiation life at the joint

The decision variable is

$$s_{12} = \frac{N_{life}}{10^6}, \quad 2 < s_{12} < 20000 \quad (34)$$

Satisfaction on s_{12} is $P(s_{12}=6.8) = 1$. Now it is by passed by setting $P = 1$ but can be activated.

This method of calculating fatigue life N_{life} combines the Haigh diagram of modified Goodman type and the S-N diagram according to (Meyer, 1985). Its is graphically shown in Figure 7

$$N_{life} = 10^A, A = \log \left[\frac{V_a V_e}{(1 - V_m) c^2} \right] \left[\frac{3}{\log(V_e / c)} \right], V_a = \frac{\sigma_{va}}{R_m}, V_m = \frac{\sigma_{vm}}{R_m}, V_e = \frac{S_e}{R_m}, c = 0.9 \quad (35)$$

Here V_a is relative effective stress amplitude, V_m is relative effective mean stress and V_e is relative effective corrected fatigue strength. Equivalent stress at the joint of A B at the upper point of the joint. The mean Mises stress and amplitude Mises stress are

$$\sigma_{V,AB} = \left[\sigma_L^2 + \sigma_T^2 + 3\tau_{LT}^2 \right]^{\frac{1}{2}}, \sigma_{Vm} = \sigma_{V,AB}, \sigma_{Va} = 0.2\sigma_{Vm} \quad (36)$$

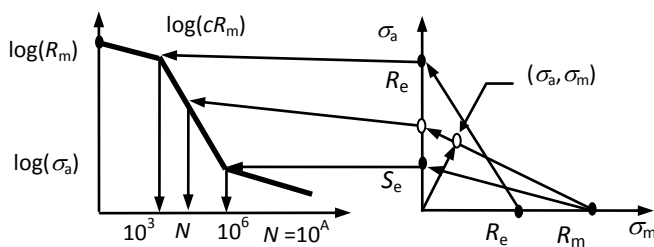


FIGURE 7: THE METHOD OF CALCULATING FATIGUE LIVES OF CRACK INITIATION TIME FROM NORMAL MEAN STRESS AND AMPLITUDE STRESS VS. S-N DIAGRAM

6.11 Goal of obtaining satisfactory fatigue crack propagation life

The decision variable is now the Paris crack propagation life prediction

$$s_{13} = \frac{N_{paris}}{10^6}, 0.1 = s_{13min} < s_{13} < s_{13max} = 10000 \quad (37)$$

The fatigue life in number of cycles from initial to final crack length is

$$N(im) = \frac{1}{C(im) \left(\frac{1}{2} m(im) - 1 \right) (\Delta \sigma_{MPa} Y \sqrt{\pi})^{m(im)}} \left[\frac{1}{a_0(im)^{\frac{1}{2}m-1}} - \frac{1}{a_f(im)^{\frac{1}{2}m-1}} \right] \quad (38)$$

6.12 Goal of obtaining satisfactory crack stress threshold safety factor

The decision variable is now safety factor against crack stress threshold exceedance. a_0 is initial crack size and a_{th} is the threshold crack size. Now satisfaction was set to 1

$$s_{14} = N_{th} = \frac{a_{th}[mm]}{a_0(im)[mm]}, a_{th0} = \frac{1}{\pi} \left(\frac{\Delta K_{th}}{Y \Delta \sigma_{max}} \right)^2 = \frac{1}{\pi} \left(\frac{2 \cdot [MPa m^{\frac{1}{2}}]}{2 \cdot 100 [MPa]} \right)^2 \approx 0.03 mm \quad (39)$$

6.13 Goals of obtaining satisfactory ecology and corrosion resistance

Goal of obtaining satisfactory ecological and corrosion resistance value depends on the material selection code im

$$s_{16} = eco(im), 0.05 < s_{16} < 0.95, s_{17} = corres(im), 0.05 < s_{17} < 0.95 \quad (40)$$

6.14 Goal of obtaining satisfactory factor of safety for the ring stiffener

The decision variable is factor of safety against stiffener ring yielding depends on the UTS of the material

$$s_{18} = N_{\text{ring}} = \frac{R_{\text{mx}}}{\sigma_{\text{max,ring}}}, R_{\text{mx}} = R_m(im) \quad (41)$$

A model of a ring stiffener on a cylinder under internal pressure is shown in Figure 8.

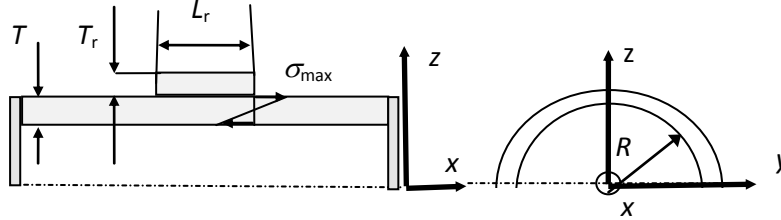


FIGURE 8: RING STIFFENER ON CYLINDER STIFFENER MODEL.

VI. RESULTS AND DISCUSSION

The aim in this section is to present the main results are depending on the pressure and material selection. The total satisfaction P is product of two satisfactions

$$P = P_{\text{tech}} \cdot P_{\text{cost}} \Rightarrow P_{\text{tech}} = \frac{P}{P_{\text{cost}}} \quad (42)$$

Figure 9 shows comparison of the results for the low and high strength steel vessels depending on pressure p .

Here R = Ratio of technical satisfactions, $R = 0.1(P_{\text{tech}}(\text{LS}) / P_{\text{tech}}(\text{HS}))$

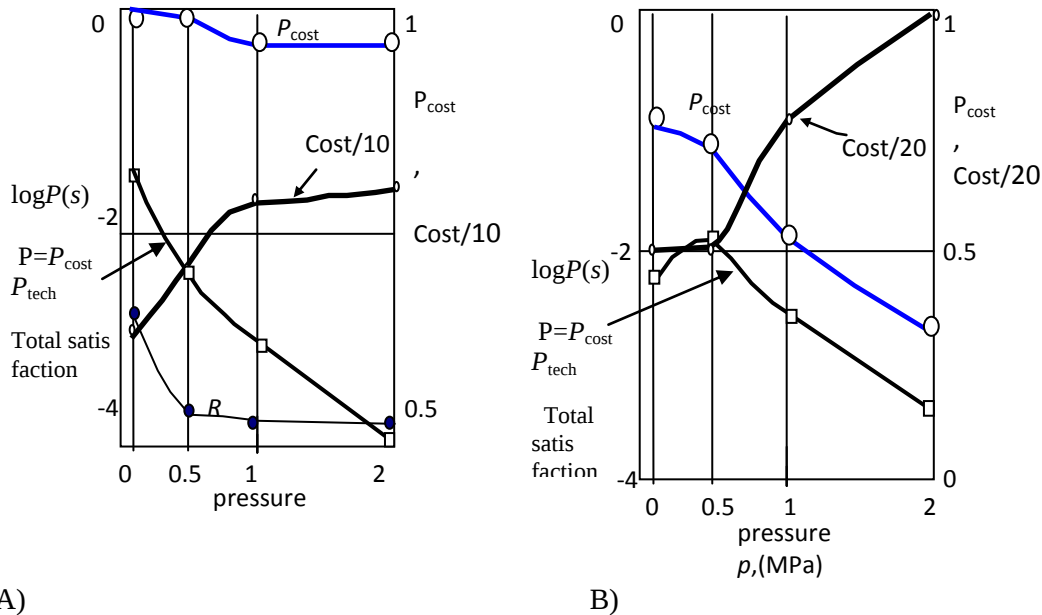


FIGURE 9: TOTAL SATISFACTION AND COST SATISFACTION AND COST OF THE VESSEL DEPENDING ON THE PRESSURE P . A) LOW STRENGTH LS B) HIGH STRENGTH HS

VII. USE OF FEM TO VERIFY OPTIMAL CONCEPTS

The results of FEM are shown in Appendix A1.

The strategy of using FEM in design was as follows. First the optimal concept to be tested was created, optimized and selected

Then the stresses and strains and buckling modes are calculated.

VIII. CONCLUSIONS

The main conclusions of the present study can be summarized as follows

A§ The motivation to this study is to develop a methodology for obtain optimal concepts vessels with tube branches

B§ The design variables are

- The material options are metal and non-metals.
- The geometrical options are wall thicknesses

C§ Using the methodology of unified fuzzy design the goals and constraints are defined by the same formulation.

- Satisfactory cost and factors of safety and other decision variables are obtained using few discrete design variable selection options

D§ Next satisfaction functions are defined for each decision variables. Then the design goal is to maximize their product.

E§ In the present study the low strength and high strength steels are competitive choices with strong and weak properties.

The total satisfaction was product of costs satisfaction and technical satisfaction.

The total satisfaction for LS steel is $P_{tot}(LS) = P_{cost}(LS) P_{tech}(LS)$

The total satisfaction for HS steel is $P_{tot}(HS) = P_{cost}(HS) P_{tech}(HS)$

At low pressure $p=0.1$ MPa $P_{tot}(LS)/P_{tot}(HS) = 4.2:1$

At high pressure $p=1$ MPa $P_{tot}(LS)/P_{tot}(HS) = 0.4:1$

Technical satisfaction behaviours differ with HS and LS vessels

- At low pressure $p = 0.1$ the ratio $P_{tech}(LS)/P_{tech}(HS) = 2.69/0.86 = 3.1:1$
- At moderate $p = 0.5$ and at $p=1$ the ratio $P_{tech}(LS)/P_{tech}(HS) = 0.5/1.7 = 0.3:1$

A possible explanation is the following

- At low pressure $p = 0.1$ MPa the strength properties of HS are underutilised and satisfaction is low
- At higher pressures the good strength properties of HS can be better utilized by using thicker walls

F§ the optimality of the concept obtained by analytical optimization is verified by FEM models.

G§ The present methodology will be used in future to explore new innovation concepts of various materials which are needed in the near future.

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APPENDIX A1. FEM RESULTS

A FEM model was made to verify the optimality of the low strength steel vessel at $p = 1\text{MPa}$ inner pressure. The FEM element model is shown in Figure A1-1.

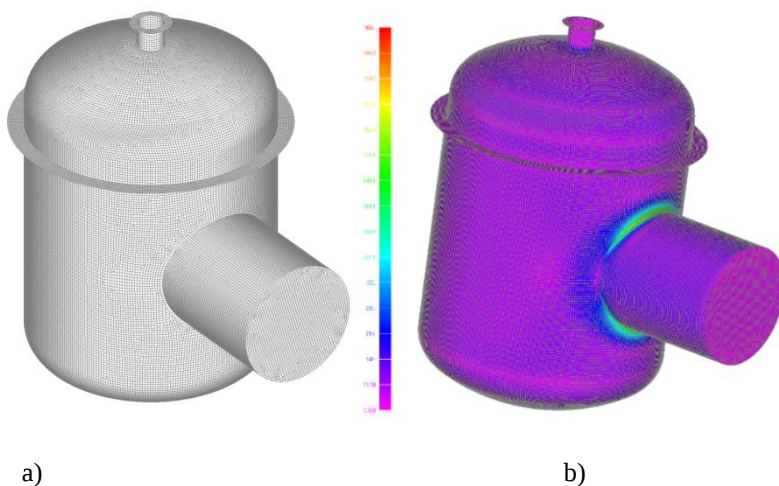


FIGURE A1-1. A) FEM MODEL SHOWING THE ELEMENTS. THERE ARE 63739 ELEMENTS AND 63624 NODES [20]. B) THE MAXIMAL STRESSES ARE AT THE UPPER AND LOWER BRANCH JOINTS

The geometry is nearly the same as in the analytical optimal model.

- vessel wall $T = 0.013$,
- branch tube wall $t_h = 0.010$ in analytical model , $t_h = 0.0050$ in FEM model
- calotte wall $T_e = 0.013$,

In the analytical model

- vessel radius $R = 1\text{m}$ half height $H=1\text{m}$,
- branch tube radius $r = 0.5\text{m}$, length $l = 1\text{m}$
- vessel calotte relative height $x_c R = c/R = 0.45$, calotte relative wall $x_e T = T_e/T=1$,
- stiffener elastic modulus $x_e E = E/E_t=1$, stiffener area $x_A R = T^2/A_t= 2$

In the FEM model nearly the same ends are used

Lower an upper elliptical ends are according the Korbboogen code.The peripheral stress in the branch tube is 103 by FEM and 100 by analytical model.