

Adaptive State Observer Synthesis Based On Instrumental Variables Method

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Abstract— The present paper presents non-recurrent adaptive observation algorithm for SISO linear time-invariant discrete systems. The algorithm is based on the instrumental variables method and the adaptive state observer estimates the parameters, the initial and the current state vectors of discrete systems. The algorithm workability is proved by using simulation data in the MATLAB/Simulink environment.

Keywords— adaptive observation, discrete adaptive state observer, non-recurrent algorithm, initial and current state vector estimation.

I. INTRODUCTION

State feedback control system design is often related to reconstruction of the state vector by measurements of the output variable and the input signal of the open loop system.

The reconstruction of the state vector is only possible by implication of state observer and the adaptive observation problem is related to observer synthesis with parameter estimator [6,7]. The matrices $\mathbf{A}$ and $\mathbf{b}$ or $\mathbf{c}$ (depending on the canonical form chosen for state space representation) are considered unknown.

The parameters are being estimated and the unknown matrices are determined during the observation process and the state vector is reconstructed.

The present paper investigates a non-recurrent algorithm for adaptive observation of single input single output (SISO) linear time invariant (LTI) discrete systems developed on the basis of the instrumental variables (IV) method [2].

The parameters estimator built-in in the adaptive observer is based on a simplified calculation procedure which also includes inversion of the informative matrix [5].

II. PROBLEM FORMULATION

The system investigated is presented in the state space with the following systems of equations:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{b}u(k), & \mathbf{x}(0) &= \mathbf{x}_0 \\ y(k) &= \mathbf{c}^T\mathbf{x}(k) + f(k) & k &= 0, 1, 2, \dots \end{aligned} \quad (1)$$

where:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \vdots & \mathbf{I}_{n-1} \\ \dots & \dots & \dots \\ \mathbf{a}^T & & \end{bmatrix} \quad (2)$$

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}; \quad \mathbf{c} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (3)$$

The system order n is a-priori known, $\mathbf{x}(k) \in R^n$ is the unknown current state vector, $\mathbf{x}(0) \in R^n$ is the unknown initial state vector, $u(k) \in R^l$ is a scalar input signal, $y(k) \in R^l$ is a scalar output signal, $f(k)$ is an additive noise signal, $\mathbf{a}$ and $\mathbf{b}$ are unknown vector parameters.

The following discrete transfer function corresponds to (1):

$$W(z) = \frac{h_1 z^{n-1} + h_2 z^{n-2} + \dots + h_{n-1} z + h_n}{z^n - a_n z^{n-1} - \dots - a_2 z - a_1} \quad (4)$$

The conformity between the elements b_i of vector $\mathbf{b}$ in relation to the chosen phase canonical form for representation and the coefficients h_i of the polynomial in the numerator of the discrete transfer function (4) are defined by the following expression[4]:

$$\mathbf{T}\mathbf{b} = \mathbf{h} \quad (5)$$

where:

$$\mathbf{h}^T = [h_1 \quad h_1 \quad \cdots \quad h_n]$$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ -a_n & 1 & \cdots & 0 & 0 \\ -a_{n-1} & -a_n & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_2 & -a_3 & \cdots & -a_n & 1 \end{bmatrix}$$

The elements a_i of vector $\mathbf{a}$, in relation to the chosen phase coordinate canonical form for representation, are the coefficients of the polynomial in the denominator of the discrete transfer function (4) presented in reversed order and with an opposite sign.

The problem to be solved is estimation of the unknown vector parameters $\mathbf{a}$ and $\mathbf{b}$, the initial state vector $\mathbf{x}(0)$ and the current state vector $\mathbf{x}(k)$, $k = 1, 2, \dots$

III. SOLUTION

3.1 Algorithm for adaptive observation based on the instrumental variables (IV) method

The developed algorithm is based on the below calculation procedure:

Step 1. Form the input-output data massive [3,8]:

$$\mathbf{u}_1 = [u(0) \quad u(1) \quad \cdots \quad u(N-2)]$$

$$\mathbf{y}_1 = [y(0) \quad y(1) \quad \cdots \quad y(N-1)]$$

$$\mathbf{y}_2 = \left[y(n) \quad y(n+1) \quad \cdots \quad y\left(\frac{N-n}{2} + n - 1\right) \right]^T$$

$$\mathbf{y}_3 = \left[y\left(\frac{N-n}{2} + n\right) \quad y\left(\frac{N-n}{2} + n + 1\right) \quad \cdots \quad y(N-1) \right]^T$$

$$\mathbf{Y}_{11} = \begin{bmatrix} -y(n-1) & -y(n-2) & \cdots & -y(0) \\ -y(n) & -y(n-1) & \cdots & -y(1) \\ -y(n+1) & -y(n) & \cdots & -y(2) \\ \vdots & \vdots & \ddots & \vdots \\ -y\left(\frac{N-n}{2} + n - 2\right) & -y\left(\frac{N-n}{2} + n - 3\right) & \cdots & -y\left(\frac{N-n}{2} - 1\right) \end{bmatrix}$$

$$\mathbf{Y}_{21} = \begin{bmatrix} -y\left(\frac{N-n}{2} + n - 1\right) & -y\left(\frac{N-n}{2} + n - 2\right) & \cdots & -y\left(\frac{N-n}{2}\right) \\ -y\left(\frac{N-n}{2} + n\right) & -y\left(\frac{N-n}{2} + n - 1\right) & \cdots & -y\left(\frac{N-n}{2} + 1\right) \\ -y\left(\frac{N-n}{2} + n + 1\right) & -y\left(\frac{N-n}{2} + n\right) & \cdots & -y\left(\frac{N-n}{2} + 2\right) \\ \vdots & \vdots & \ddots & \vdots \\ -y(N-2) & -y(N-3) & \cdots & -y(N-n-1) \end{bmatrix}$$

$$\mathbf{U}_{12} = \begin{bmatrix} u(n-1) & u(n-2) & \cdots & u(0) \\ u(n) & u(n-1) & \cdots & u(1) \\ u(n+1) & u(n) & \cdots & u(2) \\ \vdots & \vdots & \ddots & \vdots \\ u\left(\frac{N-n}{2} + n - 2\right) & u\left(\frac{N-n}{2} + n - 3\right) & \cdots & u\left(\frac{N-n}{2} - 1\right) \end{bmatrix}$$

$$U_{22} = \begin{bmatrix} u\left(\frac{N-n}{2} + n - 1\right) & u\left(\frac{N-n}{2} + n - 2\right) & \cdots & u\left(\frac{N-n}{2}\right) \\ u\left(\frac{N-n}{2} + n\right) & u\left(\frac{N-n}{2} + n - 1\right) & \cdots & u\left(\frac{N-n}{2} + 1\right) \\ u\left(\frac{N-n}{2} + n + 1\right) & u\left(\frac{N-n}{2} + n\right) & \cdots & u\left(\frac{N-n}{2} + 2\right) \\ \vdots & \vdots & \ddots & \vdots \\ u(N-2) & u(N-3) & \cdots & u(N-n-1) \end{bmatrix}$$

Where Y_{11}, Y_{21}, U_{12} and U_{22} are Toeplitz matrices and $N = 3n + 2l, l = 1, 2, 3, \dots$

Step 2. Calculate the sub-matrices $G_{11}, G_{12}, G_{21}, G_{22}$:

$$G_{11} = Y_{11}^T Y_{11} + Y_{21}^T Y_{21}; G_{12} = Y_{11}^T U_{12} + Y_{21}^T U_{22}; G_{21} = U_{12}^T Y_{11} + U_{22}^T Y_{21}; G_{22} = U_{12}^T U_{12} + U_{22}^T U_{22}$$

Step 3. Calculate the covariance matrix C :

$$C = \begin{bmatrix} M_1 + M_1 G_{12} M_2 G_{21} M_1 & \vdots & -M_1 G_{12} M_2 \\ \cdots & \cdots & \cdots \\ -M_2 G_{21} M_1 & \vdots & M_2 \end{bmatrix}$$

where:

$$M_1 = G_{11}^{-1}; M_2 = (G_{22} - G_{21} M_1 G_{12})^{-1}$$

Step 4. Calculate the vectors $\hat{h}$ and $\hat{a}$ by using the following vector-matrix system and form the estimated system matrix $\hat{A}$:

$$\hat{p} = C \begin{bmatrix} Y_{11}^T y_2 + Y_{21}^T y_3 \\ U_{12}^T y_2 + U_{22}^T y_3 \end{bmatrix}$$

$$\hat{h} = [\hat{h}_1 \quad \hat{h}_2 \quad \cdots \quad \hat{h}_n]^T = [\hat{p}_{n+1} \quad \hat{p}_{n+2} \quad \cdots \quad \hat{p}_{2n}]^T; \hat{a} = [\hat{a}_1 \quad \hat{a}_2 \quad \cdots \quad \hat{a}_n]^T = [-\hat{p}_n \quad -\hat{p}_{n-1} \quad \cdots \quad -\hat{p}_1]^T$$

$$\hat{A} = \begin{bmatrix} \mathbf{0} & \vdots & I_{n-1} \\ \cdots & \cdots & \cdots \\ \hat{a}^T & & \end{bmatrix}$$

Step 5. Calculate vector b estimation by implementing the following linear algebraic system of equations:

$$T\hat{b} = \hat{h}$$

$$\text{where: } T = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -\hat{a}_n & 1 & 0 & \cdots & 0 & 0 \\ -\hat{a}_{n-1} & -\hat{a}_n & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\hat{a}_2 & -\hat{a}_3 & -\hat{a}_4 & \cdots & -\hat{a}_n & 1 \end{bmatrix} \text{ is a lower triangular Toeplitz matrix.}$$

Step 6. Estimate the initial state vector x_0 :

$$\hat{x}_0 = (D^T D)^{-1} D^T (y_1 - Q u_1) = [\hat{x}_{01} \quad \hat{x}_{02} \quad \cdots \quad \hat{x}_{0n}]^T$$

(applicable only in case that $\det(D^T D) \neq 0$), where:

$$D = \begin{bmatrix} c^T \\ c^T \hat{A} \\ c^T \hat{A}^2 \\ \vdots \\ c^T \hat{A}^{(N-1)} \end{bmatrix}_{(N \times n)}; Q = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ c^T \hat{b} & 0 & \cdots & 0 \\ c^T \hat{A} \hat{b} & c^T \hat{b} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c^T \hat{A}^{(N-2)} \hat{b} & c^T \hat{A}^{(N-3)} \hat{b} & \cdots & c^T \hat{b} \end{bmatrix}_{(N \times (N-1))}$$

Step 7. Calculate the estimate of the output variable $y(k)$:

$$\begin{aligned} \hat{x}(k+1) &= \hat{A} \hat{x}(k) + \hat{b} u(k), & \hat{x}(0) &= \hat{x}_0 \\ \hat{y}(k) &= c^T \hat{x}(k) & k &= 0, 1, 2, \dots, N-1 \end{aligned}$$

$$\hat{F} = \hat{A} - g c^T$$

Step 8. For the instrumental matrices V_{11}, V_{21} :

$$V_{11} = \begin{bmatrix} -\hat{y}(n-1) & -\hat{y}(n-2) & \cdots & -\hat{y}(0) \\ -\hat{y}(n) & -\hat{y}(n-1) & \cdots & -\hat{y}(1) \\ -\hat{y}(n+1) & -\hat{y}(n) & \cdots & -\hat{y}(2) \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{y}\left(\frac{N-n}{2}+n-2\right) & -\hat{y}\left(\frac{N-n}{2}+n-3\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}-1\right) \end{bmatrix}$$

$$V_{21} = \begin{bmatrix} -\hat{y}\left(\frac{N-n}{2}+n-1\right) & -\hat{y}\left(\frac{N-n}{2}+n-2\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}\right) \\ -\hat{y}\left(\frac{N-n}{2}+n\right) & -\hat{y}\left(\frac{N-n}{2}+n-1\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}+1\right) \\ -\hat{y}\left(\frac{N-n}{2}+n+1\right) & -\hat{y}\left(\frac{N-n}{2}+n\right) & \cdots & -\hat{y}\left(\frac{N-n}{2}+2\right) \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{y}(N-2) & -\hat{y}(N-3) & \cdots & -\hat{y}(N-n-1) \end{bmatrix}$$

Step 9. Recalculate the submatrices G_{11} and G_{12} :

$$G_{11} = V_{11}^T Y_{11} + V_{21}^T Y_{21}; G_{12} = V_{11}^T U_{12} + V_{21}^T U_{22}$$

Step 10. Recalculate the parameters vector p :

$$M_1 = G_{11}^{-1}; M_2 = (G_{22} - G_{21} M_1 G_{12})^{-1}$$

$$C = \begin{bmatrix} -M_2 G_{22} M_1 & \vdots & M_2 \\ \cdots & \cdots & \cdots \\ M_1 + M_1 G_{11} M_2 G_{22} M_1 & \vdots & -M_1 G_{11} M_2 \end{bmatrix}; \hat{p} = C \begin{bmatrix} V_{11}^T y_2 + V_{21}^T y_3 \\ U_{12}^T y_2 + U_{22}^T y_3 \end{bmatrix}$$

$$\hat{h} = [\hat{h}_1 \quad \hat{h}_2 \quad \cdots \quad \hat{h}_n]^T = [\hat{p}_{n+1} \quad \hat{p}_{n+2} \quad \cdots \quad \hat{p}_{2n}]^T; \hat{a} = [\hat{a}_1 \quad \hat{a}_2 \quad \cdots \quad \hat{a}_n]^T = [-\hat{p}_n \quad -\hat{p}_{n-2} \quad \cdots \quad -\hat{p}_1]^T$$

$$\hat{A} = \begin{bmatrix} 0 & \vdots & I_{n-1} \\ \cdots & \cdots & \cdots \\ \hat{a}^T & & \end{bmatrix}$$

Step 11. Repeat steps 7 to 10 four times

Step 12. Estimate the current state vector $x(k)$:

$$\hat{x}(k+1) = \hat{F}\hat{x}(k) + \hat{b}u(k) + g y(k), \hat{x}(0) = \hat{x}_0; \hat{F} = \hat{A} - g c^T$$

Vector g can be easily obtained by solving the pole placement problem (PPP) also known as pole assignment problem (PAP). The synthesis of vector g must take into consideration the following options: the eigen values of the matrix $\hat{F}$ should be zeros or should be spread into the unit circle closer to the origin than the matrix $\hat{A}$ eigenvalues. The implementation of the above listed options ensures good dynamic characteristics of the observer synthesized.

IV. SIMULATION RESULTS

The simulation is held in MATLAB programming environment under the below conditions:

- For a given transfer function of the system under investigation, with input signal $u(k)$ and the respective output signal $y(k)$;
- Added colored noise signal $f(k)$ is applied to the system output;
- The input signal $u(k)$ and the noise-corrupted output signal $y(k)$ are used as an input data for the observation algorithm;
- Based on the input-output data massive, the algorithm developed calculates the estimates of the open loop system and the state vector.

The discrete transfer function of the system investigated, used for the simulations is presented as follows:

$$W(z) = \frac{0.6z^{-1} + 0.56z^{-2} + 0.2125z^{-3} + 0.308z^{-4} + 0.5488z^{-5} + 0.7221z^{-6}}{1 - 1.4z^{-1} + 0.7875z^{-2} - 0.2275z^{-3} + 0.035525z^{-4} - 0.002835z^{-5} + 0.00009z^{-6}}$$

and its corresponding representation in state space:

$$\mathbf{a} = \begin{bmatrix} -0.00009 \\ 0.002835 \\ -0.035525 \\ 0.2275 \\ -0.7875 \\ 1.4 \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 0.6 \\ 0.2 \\ 0.1 \\ 0.3 \\ 0.4 \\ 0.5 \end{bmatrix}; \mathbf{c} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

The matrix $\mathbf{A}$ eigenvalues are obtained by using $\text{eig}(\cdot)$ function in MATLAB:

$$\text{eig}(\mathbf{A}) = [0.4 \quad 0.3 \quad 0.25 \quad 0.2 \quad 0.15 \quad 0.1]^T$$

Pseudo-random binary sequence (PRBS) is used as an input signal $u(k)$ which is generated in MATLAB by using the following functions: $u = (\text{sign}(\text{randn}(127,1))) * 10$.

The output signal $y(k)$ is noise-corrupted by adding a color noise $f(k)$. The colored noise is obtained by filtering of white noise through filter with the following transfer function:

$$W_f(z) = \frac{1}{1 - 1.4z^{-1} + 0.7875z^{-2} - 0.2275z^{-3} + 0.035525z^{-4} - 0.002835z^{-5} + 0.00009z^{-6}}$$

The noise level η is calculated mathematically by division of the noise standard deviation σ_f to the output signal standard deviation σ_y in accordance with the equation given below:

$$\eta = \frac{\sigma_f}{\sigma_y} \cdot 100 = (0 \div 10)\%$$

Vector $\mathbf{a}$ estimation error e_a , vector $\mathbf{b}$ estimation error e_b and the state vector $\mathbf{x}(k)$ estimation error e_x are relative mean squared errors (RMSE) and could be determined by the following equations:

$$e_a(k) = -\sqrt{\frac{\sum_{i=1}^n (a_i(k) - \hat{a}_i(k))^2}{\sum_{i=1}^n a_i(k)}}; e_b(k) = -\sqrt{\frac{\sum_{i=1}^n (b_i(k) - \hat{b}_i(k))^2}{\sum_{i=1}^n b_i(k)}}; e_x(k) = -\sqrt{\frac{\sum_{i=1}^n (x_i(k) - \hat{x}_i(k))^2}{\sum_{i=1}^n x_i(k)}}$$

The results for the case of noise-free output signal (i.e. $f(k) = 0$) and $l = 0$ (i.e. minimum number of the input-output measurements: $N = 3n = 18$) are shown in Fig.1. In accordance with the initial settings the algorithm will start working at the 18th step of the calculations and in this case in particular the observation errors $e_a(k)$, $e_b(k)$ and $e_x(k)$ are zeros.

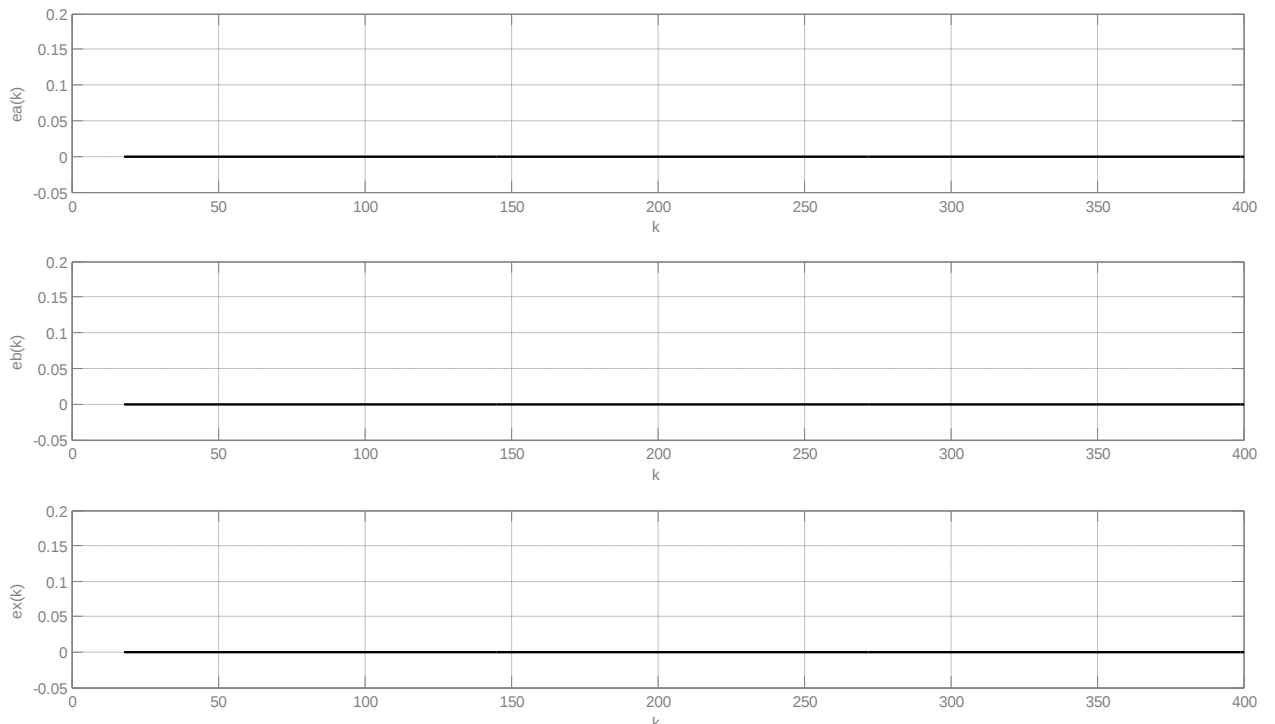


FIGURE 1. RMSE FOR THE CASE OF NOISE-FREE OUTPUT SIGNAL

In the case of noise-corrupted output signal an experiment is carried out for noise level $\eta = 10.018\%$ and $l = 40$ (i.e. $N = 3n + 2l = 98$). The results are shown in Fig.2. Under the above described initial settings the algorithm will start working at the 98th step of the calculations and the RMSE are as follows: $e_a(k) < 0.033$, $e_b(k) < 0.010$, $e_x(k) < 0.065$.

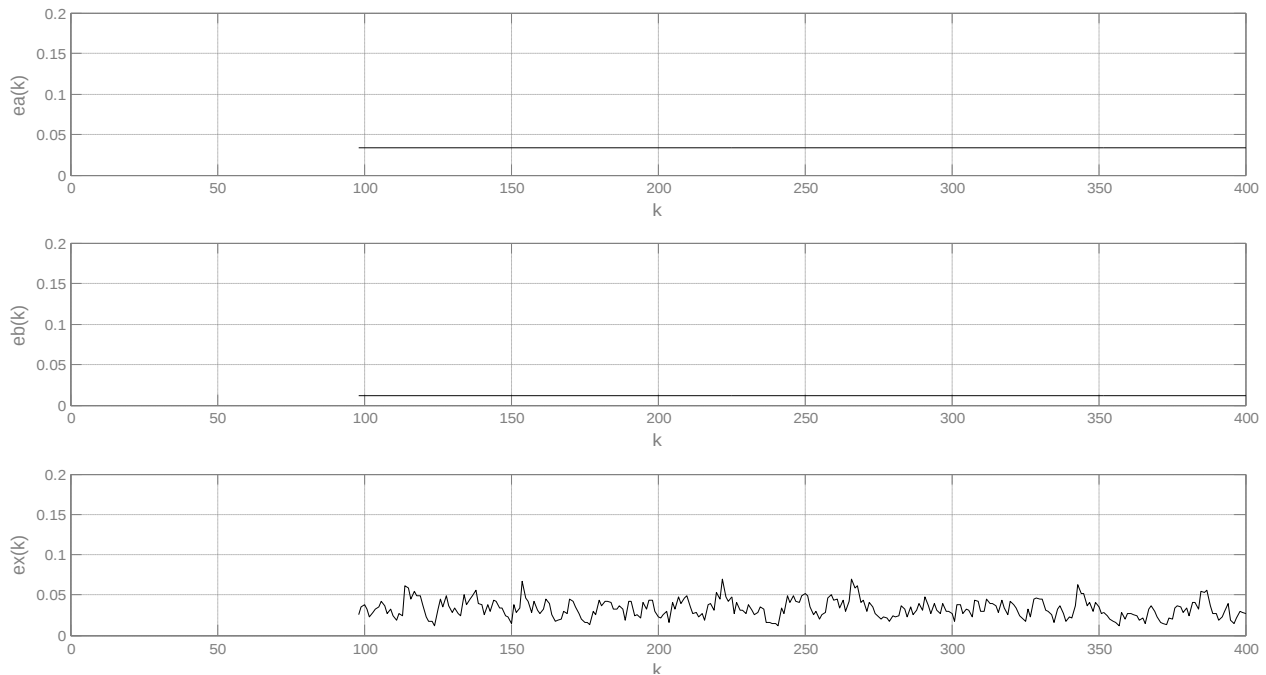


FIGURE 2. RMSE FOR THE CASE OF NOISE-CORRUPTED OUTPUT SIGNAL, $\eta = 10.018\%$, $N = 3n + 2l = 98$

An additional experiment is carried out for the provisions of noise-corrupted output signal analysis for noise level $\eta = 10.014\%$ and $l = 100$ (i.e. $N = 3n + 2l = 218$). The results show that the algorithm starts working at the 218th step of the calculations and the RMSE are as follows: $e_a(k) < 0.017$, $e_b(k) < 0.0057$, $e_x(k) < 0.024$, under the condition: $218 < k < 400 \Rightarrow e_x(k) = 0.01$ (see Fig.3).

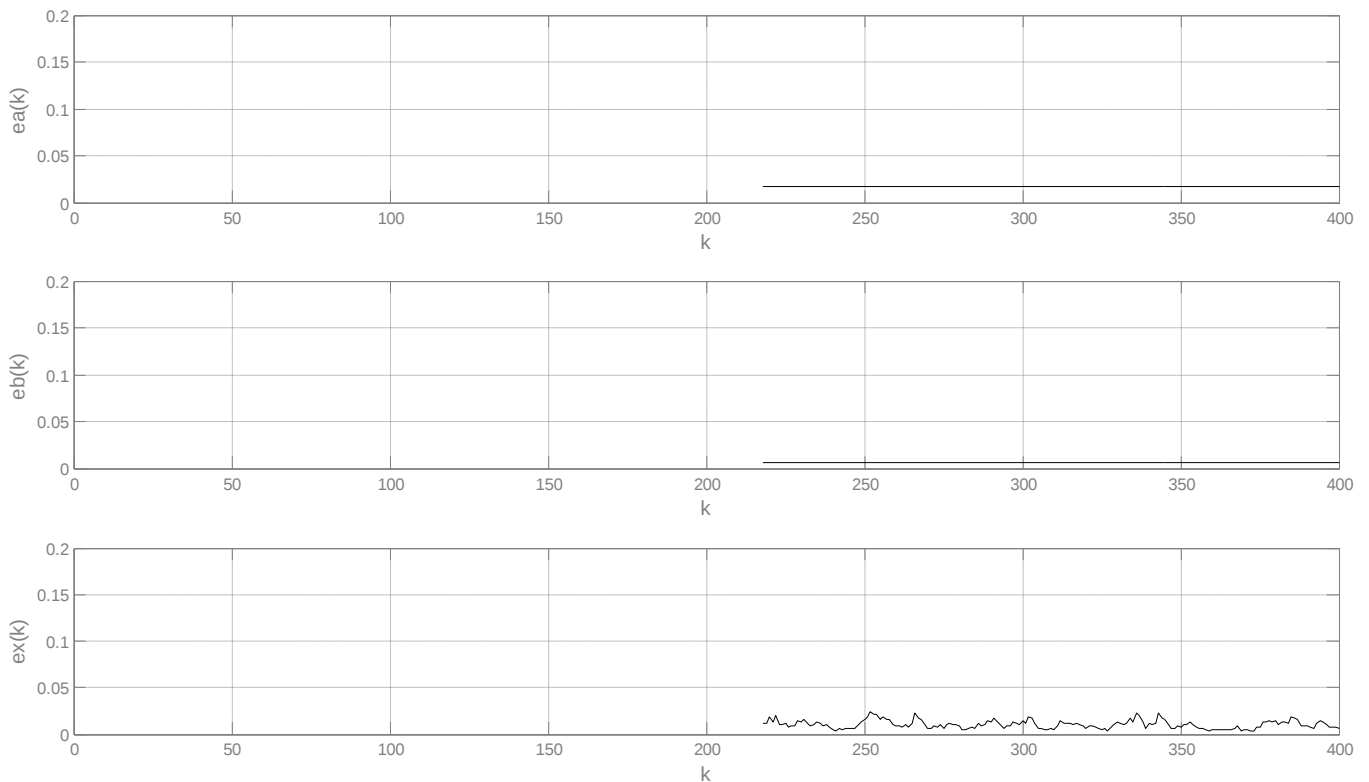


FIGURE 3. RMSE FOR THE CASE OF NOISE-CORRUPTED OUTPUT SIGNAL, $\eta = 10.014\%$, $N = 3n + 2l = 218$

The simulation results delivered and graph analysis show that with the increase in number of the input-output measurements (N), the algorithm invariance to added noises increases proportionally however the time needed for collection of initial information rises.

V. CONCLUSIONS

The algorithm suggested for open loop system parameter estimations which serve as a basis for further reconstruction of the current state vector implements the method of the instrumental variables excluding the zero iteration which only uses the least squares method (steps 1 to 4 of the suggested calculation procedure).

The algorithm proposed estimates as well the initial state vector $\mathbf{x}_0$ which allows the forming of the instrumental variables matrix even for nonzero initial conditions.

The results delivered show that the number of the input output data measurements (N) is of high significance in relation to accuracy of estimations in the case of noise corrupted output. The highest accuracy is to be expected for highest counts of N (see Fig.2 and Fig.3).

The method of the IV method gives best results in case of estimation of a-priori collection of data [3,8], however in relation to the closed loop system the added noise $f(k)$ is transferred to the input signal through the feedback channel. Thus invariance between the instrumental matrices and the added noise is not possible; it is only possible that the estimates are unbiased and significant in presence of a white noise however the real systems do not allow such solution of the problem. The used of IV method for investigation of the closed loop system is only applicable if additional input signal is implemented [8]. For this reason the implementation of the algorithm suggested above is not recommended for closed systems implications.

The algorithm for adaptive observation based on the instrumental variables (IV) method introduced in the present paper is developed on the basis of non-recurrent method which ensures the convergence of the iterative procedure [1,2].

The most positive feature of this algorithm however is related to the method used for informative matrix formation. It is formed through the four sub-matrices $\mathbf{Y}_{11}$, $\mathbf{Y}_{21}$, $\mathbf{U}_{12}$ and $\mathbf{U}_{22}$ which reduces the calculation complexity of the procedure for inversion of matrix $\mathbf{G}$ formed by the sub-matrices $\mathbf{G}_{11}$, $\mathbf{G}_{12}$, $\mathbf{G}_{21}$, $\mathbf{G}_{22}$. Independently of the N count in numbers for the estimation of the coefficients h_i and a_i is only needed to be inverted the matrices $\mathbf{G}_{11}$ and $(\mathbf{G}_{22} - \mathbf{G}_{21}\mathbf{M}_1\mathbf{G}_{12})$ which are always guaranteed $(n \times n)$ dimensions. In all other cases this procedure is related to inversion of a matrix at least $(N - n) \times (N - n)$ dimensional.

ACKNOWLEDGMENT

The paper is developed in the frames of the project HPI6 "Research and Synthesis of Algorithms and Systems for Adaptive Observation, Filtration and Control", ДН997-НП/09.05.2017.

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